

### CATALOG DESCRIPTION

Instructs the student in the techniques of multivariable calculus. Topics include partial differentiation, linear and quadratic approximations, optimization, multiple integration, vector fields, line and flux integrals, curl, divergence and the three fundamental theorems. Mathematical software will be utilized throughout the course to expose students to computer algebra systems.

Prerequisites: MATH 189 with a grade of "C" or better.

Semester Offered: Fall and Spring

#### Common Student Learning Outcomes

Upon successful completion of San Juan College programs and degrees, the student will demonstrate competency in...

#### BROAD AND SPECIALIZED LEARNING

Students will actively and independently acquire, apply, and adapt skills and knowledge with an awareness of global contexts.

#### CRITICAL THINKING

Students will think analytically and creatively to explore ideas, make connections, draw conclusions and solve problems.

#### CULTURAL AND CIVIC ENGAGEMENT

Students will act purposefully, reflectively, and ethically in diverse and complex environments.

#### EFFECTIVE COMMUNICATION

Students will exchange ideas and information with clarity in multiple contexts.

#### INFORMATION LITERACY

Students will be able to recognize when information is needed and have the ability to locate, evaluate, and use it effectively.

#### INTEGRATING TECHNOLOGIES

Students will demonstrate fluency in the application and use of technologies in multiple contexts.

Student work from this class may be randomly selected and used anonymously for assessment of course, program, and/or institutional learning outcomes. For more information, please refer to the Dean of the appropriate School.

#### **General Learning Outcomes**

Upon successful completion of the course, the student will be able to...

- A. Functions of many variables.
- B. The tools of vector analysis.
- C. Differentiation of functions of many variables.
- D. Methods of optimization.
- E. Integration of functions of many variables.
- F. Parametric equations in a multivariable setting.
- G. Vector fields and line integrals.

H. The calculus of vector fields.

### **Specific Learning Outcomes**

Upon successful completion of the course, the student will be able to...

### A. Functions of Many Variables

- A1. Understand and be able to describe the properties of functions of two variables.
- A2. Be able to graph basic functions in three-dimensional space, graph general functions in two variables, and graph linear functions of two variables.
- A3. Understand and be able to draw contour diagrams.
- A4. Be able to analyze planes in a three-dimensional context.
- A5. Understand and be able to visualize level surfaces and graphs of implicitly defined functions (cylinders, quadric surfaces, etc.) and linear functions of more than two variables.

### B. Vector Analysis

- B1. Understand the definition of a vector and position vectors in three-dimensional space and be able to describe vectors geometrically and numerically.
- B2. Be able to add and subtract vectors, multiply vectors by scalars, use vector properties, and find dot and cross products of vectors. Understand the geometric meaning of the results of these operations.
- B3. Be able to find the magnitude and direction of vectors.
- B4. Be able to find the angle between two vectors and determine whether they are parallel or orthogonal.
- B5. Be able to use vectors to find area and volume.
- B6. Understand the definition of a vector valued function. Be able to find the limit and the domain of vector valued functions.
- B7. Be able to determine whether a vector function is continuous and find the parametric equations for the associated space curve.
- B8. Be able to find derivatives and integrals of vector-valued functions. Be able to use the derivative to find tangent lines and tangent vectors to space curves.
- B9. Be able to find arc length of a vector function or space curve.
- B10. Be able to find velocity and acceleration vectors and the speed of a particle moving along a space curve, including tangential and normal components of acceleration.

# C. Differentiation of Functions of Many Variables

- C1. Understand the definition of limit for a multivariable function. Be able to determine a limit or show that a limit does not exist by calculating limits on different paths.
- C2. Understand what it means for a function to be continuous and be able to determine whether a function of two or three variables is continuous.
- C3. Be able to compute partial derivatives of a function of several variables and understand the geometric interpretation of partial derivatives. Know Clairaut's Theorem.
- C4. Be able to find the tangent planes to surfaces in three-dimensional space.
- C5. Understand linearization as an approximation of tangent planes and be able to differentiate to determine local linearity.
- C6. Understand the definition of the directional derivative and be able to find directional

derivatives and gradients.

- C7. Understand and be able to use the chain rule to compute derivatives of composite functions in multiple variables.
- C8. Be able to find derivatives implicitly using the partial derivatives.
- C9. Be able to find higher order partial derivatives.
- C10. Be able to find quadratic approximations for functions of two variables.

## D. Methods of Optimization

- D1. Be able to use directional derivatives to find the maximum rate of change.
- D2. Understand the definition and be able to find local and absolute maximum and minimum values and saddle points of functions of two variables. Be able to find critical points.
- D3. Be able to solve practical problems involving constrained (Lagrange Multipliers) and unconstrained optimization.

## E. Integration of Functions of Many Variables

- E1. Understand the definition of double and triple integrals as the limit of Riemann sums.
- E2. Be able to set up and evaluate double integrals in rectangular and polar coordinates.
- E3. Be able to evaluate which coordinate system is most appropriate for a given problem and convert from rectangular to polar coordinates and vice versa, as necessary.
- E4. Know Fubini's Theorem for double and triple integrals.
- E5. Be able to set up and evaluate triple integrals in rectangular, cylindrical and spherical coordinates.
- E6. Be able to evaluate which coordinate system is most appropriate for a given problem and convert from rectangular to either cylindrical or spherical coordinates and vice versa, as necessary.
- E7. Be able to find area and volume using double or triple integrals, as appropriate. Be able to switch limits of integration to facilitate evaluating integrals.
- E8. Be able to use a Jacobian to evaluate a change of variables in a double integral.
- E9. Be able to use double and triple integrals in applications problems.

## F. Parametric Equations in a Multivariable Setting

- F1. Be able to solve parametric equations in two and three dimensions.
- F2. Be able to analyze lines in a three-dimensional context.
- F3. Be able to solve problems involving motion, velocity and acceleration and the speed of a particle moving along a space curve.

## G. Vector Fields and Line Integrals

- G1. Understand the definition of a vector field in two- and three-dimensional space.
- G2. Understand and be able to find a gradient vector field.
- G3. Be able to evaluate vector fields and the flow of a vector field.
- G4. Be able to find the vector equation for parameterized surfaces in two and three dimensions.
- G5. Understand the definition of a line integral and be able to find the line integral of a function on a smooth plane or space curve.
- G6. Understand and be able to apply the Fundamental Theorem of Line Integrals.

- G7. Understand what is meant by independence of path, and be able to determine when a line integral has this property.
- G8. Understand the definition of conservative and be able to determine whether a vector field is conservative. Be able to find the potential function (given a conservative vector field).
- G9. Understand and be able to apply Green's Theorem.

### H. The Calculus of Vector Fields

- H1. Understand and be able to find curl and divergence of a vector field.
- H2. Understand and be able to use the Divergence Theorem.
- H3. Be able to calculate flux integrals, specifically flux integrals for graphs, cylinders, and over parameterized surfaces.
- H4. Understand and be able to use Stokes's theorem.
- H5. Be able to calculate the surface area of a surface, whether given parametrically or explicitly. Be able to set up and evaluate the surface integral of a function over a surface.

#### **Other Requirements:**

The TI-82, TI-83, TI-84, TI-85 or TI-86 graphing calculator is required for the course (TI-Nspire calculators that are equivalent to these are acceptable). A **TI-83 Plus or TI-84 Plus Graphing Calculator** is strongly recommended. Graphing calculators capable of symbolic manipulation (such as TI-89, TI-92, TI-Nspire CAS systems and other such calculators) will not be allowed on examinations, the final exam and where the instructor sees fit.